Ratios

A ratio is a way of comparing two or more similar quantities, by writing two or more numbers separated by colons. The numbers should be whole numbers, and should not include units.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- calculate the ratio of two or more similar quantities, whether or not they are expressed in the same units;
- divide a quantity into a number of parts in given ratios;
- use ratios to scale up, or scale down, a list of ingredients.

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1. Introduction

A ratio is a way of comparing two or more similar quantities. Ratios can be used to compare costs, weights, sizes and other quantities.

For example, suppose we have a model boat which is 1 m long, whereas the actual boat is 25 m long. Then the ratio of the length of the model to the length of the actual boat is 1 to 25. This is written as

$$1 : 25\,.$$

Note there are no units included, and note also the use of the colon to represent the ratio.

Ratios are also used to describe quantities of different ingredients in mixtures. Pharmacists making up medicines, manufacturers making biscuits and builders making cement all need to make mixtures using ingredients in the correct ratio. If they don't there may be dire consequences! So knowing about ratios is not only very important, but extremely useful and crucial in certain circumstances.

For example, mortar for building a brick wall is made by using 2 parts of cement to 7 parts of sand. Then the ratio of cement to sand is 2 to 7, and is written as

$$2 : 7\,.$$

2. Simplifying ratios

To make pastry for an apple pie, you need 4 oz flour and 2 oz fat. The ratio of flour to fat is

$$4 : 2\,.$$

But this ratio can be simplified in the same way that two fractions can be simplified. We just cancel by a common factor. So

$$4 : 2 = 2 : 1\,.$$

The ratio 2 to 1 is the simplest form of the ratio 4 to 2. And the ratios are equivalent, because the relationship between each pair of numbers is the same.

For example, if we have a ratio 250 to 150, we can simplify it by dividing both numbers by 10 and then by 5 to get 5 to 3:

$$\begin{gathered} 250 : 150 \\ 25 : 15 \\ 5 : 3\,. \end{gathered}$$

The ratio 5 to 3 is the simplest form of the ratio 250 to 150, and all three ratios are equivalent.

Ratios are normally expressed using whole numbers, so a ratio of 1 to 1.5 would be written as 10 to 15, and then as 2 to 3 in its simplest form:

$$\begin{gathered} 1 : 1.5 \\ 10 : 15 \\ 2 : 3\,. \end{gathered}$$

Similarly, a ratio $\frac{1}{4}$ to $\frac{5}{8}$ would be written as $\frac{2}{8}$ to $\frac{5}{8}$, and then as 2 to 5 in its simplest form:

$$\begin{aligned}\tfrac{1}{4} &: \tfrac{5}{8}\\ \tfrac{2}{8} &: \tfrac{5}{8}\\ 2 &: 5\ .\end{aligned}$$

Now it is very important in a ratio to use the same units for the numbers, as otherwise the ratio will be incorrect and the comparison will be wrong. Take this ratio: 15 pence to £3. The ratio is not 15 to 3 and then 5 to 1. The comparison is wrong. We must have the same units for each number, so we convert them to the same units. It doesn't matter which unit you use, but of course it is just use common sense to choose the unit which gives the simplest numbers. In this case it is obvious that we should use pence, so 15 pence to 300 pence is then simplified to 3 to 60 by dividing by 5. We then simplify it further by dividing by 3 to get 1 to 20. That is the ratio in its simplest form. So

$$\begin{aligned}15\,\text{p} &: \pounds 3\\ 15 &: 3\\ 5 &: 1\end{aligned}$$

is wrong, whereas

$$\begin{aligned}15\,\text{p} &: \pounds 3\\ 15 &: 300\\ 3 &: 60\\ 1 &: 20\end{aligned}$$

is correct.

Key Point

A ratio is a way of comparing two or more similar quantities. A ratio of 2 cm to 5 cm is written as 2 : 5. A ratio is normally written using whole numbers only, with no units, in its simplest form.

The numbers in a ratio must be written using the same units. If they are not, they should be converted to the same units. It does not matter which units are used for the conversion.

Exercises

1. Express these ratios in their simplest form:

(a)	2 to 10	(b)	80 to 20	(c)	$\frac{1}{3}$ to 1	(d)	50 p : £3.50
(e)	6 m : 30 cm	(f)	1.5 : 1	(g)	10 min : 4 hr	(h)	$\frac{4}{3}$: 3

2. In a class there are 15 girls and 12 boys. What is the ratio of girls to boys?

3. Anna has 75 pence. Rashid has £1.20. What is the ratio of Rasid's money to Anna's money?

3. Using ratios to share quantities

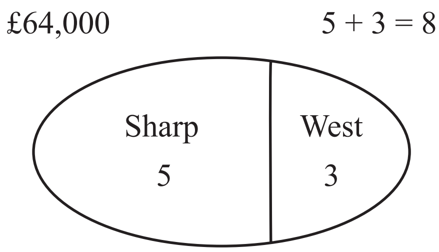
Ratios can be used to share, or divide, quantites of money, weights and so on.

Example

Mrs Sharp and Mr West share an inheritance of £64,000 in the ratio 5 : 3. How much do they each get?

Solution

To calculate the answers we first look at the numbers involved and see the total number of parts into which the inheritance is split. The ratio is 5 to 3. So the total number of parts is 5 plus 3, which is 8.



Now we can work out what one part is worth, and then how much each person gets.

$$\begin{aligned} 1 \text{ part} &= \frac{£64,000}{8} \\ &= £8,000. \end{aligned}$$

So Mrs Sharp receives 5 parts, which is $5 \times £8,000 = £40,000$ and Mr West receives 3 parts, which is $3 \times £8,000 = £24,000$.

We can check our calculations by adding the two amounts together. They should add up to the total value of the inheritance. So

$$£40,000 + £24,000 = £64,000$$

which does equal the original inheritance.

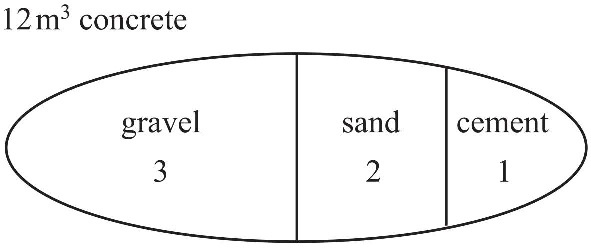
We can also check this calculation in another way. We can work backwards, by taking the two amounts and finding their ratio. The two amounts are both given in the same units, pounds, and so

$$\begin{gathered} 40,000 : 24,000 \\ 40 : 24 \\ 5 : 3\ . \end{gathered}$$

Example

Concrete is made by mixing gravel, sand and cement in the ratio 3 : 2 : 1 by volume. How much gravel will be needed to make $12\,\mathrm{m}^3$ of concrete?

Solution



First, we work out the total number of parts into which the concrete is divided: $3 + 2 + 1 = 6$ parts altogether. Using the numbers in the ratio, we know then that gravel makes up 3 parts, sand 2 parts, and cement 1 part. So there are 6 parts altogether, and we have $12\,\mathrm{m}^3$ of concrete, and therefore 1 part must equal $2\,\mathrm{m}^3$. Then as there are 3 parts of gravel, the volume of gravel needed must be $3 \times 2\,\mathrm{m}^3$ which is $6\,\mathrm{m}^3$:

$$\begin{aligned} 3+2+1 &= 6 \text{ parts} \\ 6 \text{ parts} &= 12\,\mathrm{m}^3 \\ 1 \text{ part} &= \tfrac{12}{6}\,\mathrm{m}^3 \\ &= 2\,\mathrm{m}^3 \\ \text{so gravel (3 parts)} &= 3 \times 2\,\mathrm{m}^3 \\ &= 6\,\mathrm{m}^3. \end{aligned}$$

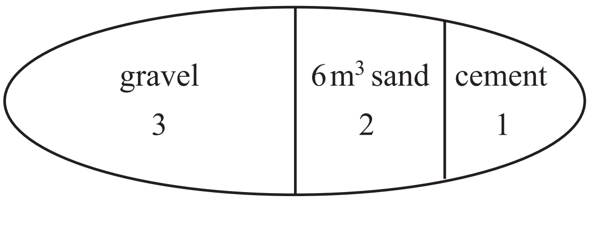
We now need to check the answer. Gravel represents 3 parts out of a total of 6, in other words a half. So half of the total volume of concrete is gravel, and that is half of $12\,\mathrm{m}^3$, which is $6\,\mathrm{m}^3$. So that is indeed the correct answer.

Example

With the same formula for concrete, suppose we have $6\,\mathrm{m}^3$ of sand and an unlimited amount of the other ingredients. How much concrete could we make?

Solution

In this example, the ratio of gravel to sand to cement is still $3 : 2 : 1$, so the total number of parts into which the concrete is divided is still $3+2+1 = 6$. But this time we know the volume of sand, and we have to work out the total volume of concrete that is possible to make.



Two parts of the total represents $6\,\mathrm{m}^3$ of sand. So one part is $\frac{6}{2}\,\mathrm{m}^3$, in other words $3\,\mathrm{m}^3$, and thus the total of 6 parts of concrete represents $3 \times 6\,\mathrm{m}^3$, making $18\,\mathrm{m}^3$. So $18\,\mathrm{m}^3$ of concrete can be made if we have $6\,\mathrm{m}^3$ of sand and an unlimited amount of the other ingredients:

$$\begin{aligned} 3+2+1 &= 6 \text{ parts} \\ 2 \text{ parts} &= 6\,\mathrm{m}^3 \\ 1 \text{ part} &= \tfrac{6}{2}\,\mathrm{m}^3 \\ &= 3\,\mathrm{m}^3 \\ \text{so } 6 \text{ parts} &= 6\times 3\,\mathrm{m}^3 \\ &= 18\,\mathrm{m}^3\,. \end{aligned}$$

Alternatively, we could have tackled this question by using fractions. Sand represents 2 parts out of a total of 6, which is a third. So if a third of the total is $6\,\mathrm{m}^3$ then the total amount of concrete that could be made would be 3 times 6, giving $18\,\mathrm{m}^3$. This is a good check that our answer is correct.

Example Here is a list of the ingredients to make a quantity of the Greek food houmous sufficient for 6 people.

2 cloves garlic
4 oz chick peas
4 tbs olive oil
5 fl oz tahini paste (houmous for 6 people)

What amounts would be needed so that there will be enough for 9 people?

Solution
The ratio of the amounts is 2 : 4 : 4 : 5 for 6 people. For one person we scale the amounts down, so we divide by 6. Then for 9 people we multiply by 9, and we see after cancelling that we need 3 cloves of garlic, 6oz chick peas, 6 tbs of olive oil, and $7\frac{1}{2}$ fl oz of tahini paste:

$$\begin{array}{ccccccccl} 2 & : & 4 & : & 4 & : & 5 & & \text{(6 people)} \\ \frac{2}{6} & : & \frac{4}{6} & : & \frac{4}{6} & : & \frac{5}{6} & & \text{(1 person)} \\ \frac{1}{3} & : & \frac{2}{3} & : & \frac{2}{3} & : & \frac{5}{6} & & \\ \frac{1}{3}\times 9 & : & \frac{2}{3}\times 9 & : & \frac{2}{3}\times 9 & : & \frac{5}{6}\times 9 & & \text{(1 person)} \\ 3 & : & 6 & : & 6 & : & \frac{15}{2}=7\frac{1}{2} & & \end{array}$$

giving

3 cloves garlic
6 oz chick peas
6 tbs olive oil
$7\frac{1}{2}$ fl oz tahini paste (houmous for 9 people).

We could have done these calculations more quickly by multiplying each amount by the fraction 9/6, or 3/2 in its simplest form. But it is often safer to work out what the amounts are for one person, and then scale up or down afterwards accordingly.

In conversion problems, it is often better to work out what one of the required amounts represents, and then scale up or down.

Example
If £1 is worth 1.65 euros, what is the value of 50 euros to the nearest penny?

Solution

We are given that 1.65 euros is worth £1 or 100 pence, so 1 euro is worth 100/1.65 pence. Then 50 euros equals 100/1.65 times 50 pence, which is 5000/1.65 pence. Putting this into a calculator gives 3030.3030, which is 3030 pence to the nearest penny, or 30.30. So

$$\begin{aligned}
1.65 \text{ euros} &= £1 \\
&= 100 \text{ pence} \\
1 \text{ euro} &= \frac{100}{1.65} \text{ pence} \\
50 \text{ euro} &= \frac{100}{1.65} \times 50 \text{ pence} \\
&= \frac{5000}{1.65} \text{ pence} \\
&= 3030 \text{ pence} \\
&= £30.30\,.
\end{aligned}$$

Key Point

When dividing a quanity in a given ratio, it is useful to work out

- the total number of parts into which the quantity is to be divided, and
- the value of one part.

Exercises

4. A map scale is 1 : 20,000. On the map, the distance between two points X and Y is 8.5 cm. What is the actual distance between X and Y?

5. Arminder does a scale drawing of his living room. He uses a scale of 1 : 100. He measures the length of the living room as 13.7 m. How long is the living room on the scale drawing?

6. A recipe to make lasagna for 5 people uses 300 grams of minced beef. How much minced beef would be needed to serve 9 people?

7. The ratio of boys to girls in a youth club is 4 : 5. There are 28 boys. How many girls are there in the youth club?

8. One pound is worth 1.65 euros.

- What is 20 pounds in euros?
- What is 60 euros to the nearest penny?

9. Betty is 12 years old and her sister Liz is 3 years old. Their grandfather gives them £150, which is to be divided between them in the ratio of their ages. How much does each of them get?

10. Divide 360° into three angles in the ratio 1 : 2 : 3.

11. Blue copper sulphate is made from

32 parts of copper
16 parts of sulphur
32 parts of oxygen
48 parts of water

where all the proportions are by weight.

- How much water is there in 5 kg of copper sulphate?
- How much copper sulphate could be made with 96 kg of copper and plenty of all other ingredients?

12. Here are the ingredients for making 18 rock cakes:

9 oz flour
6 oz sugar
6 oz margarine
8 oz mixed dried fruit
2 large eggs.

- Robert wants to make 12 rock cakes. How much margarine does he need?
- Jenny has only 9 oz of sugar and has plenty of all the other ingredients. What is the greatest number of rock cakes she can make?

Answers

1.

(a)	1 : 5	(b)	4 : 1	(c)	1 : 3	(d)	1 : 7	(e)	20 : 1
(f)	3 : 2	(g)	1 : 24	(h)	4 : 9				

2. 5 : 4

3. 8 : 5

4. 1700 m

5. 13.7 cm

6. 540 gm

7. 35

8. (a) 33 euros (b) £36.36, or 3636 pence

9. Betty receives £120, Liz receives £30

10. 60°, 120° and 180°

11. (a) 1875 gm (or 1.875 kg) (b) 384 kg

12. (a) 4 oz (b) 27